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Register Number:

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**ST. JOSEPH’S COLLEGE (AUTONOMOUS), BANGALORE – 27**

M.Sc. STATISTICS – I SEMESTER

SEMESTER EXAMINATION: OCTOBER 2021

(Examination conducted in February-March 2022)

 **ST 7121 - Probability Theory**

**Time: 2 ½ hrs Max. Marks-70**

This question paper has **TWO** printed pages and **TWO** parts

**PART A**

**Answer any SIX of the following 3x6=18**

1. Find the limit of a sequence of sets {$A\_{n}:n\geq 1$}, where $A\_{n}=\left(2-\frac{1}{n},2+\frac{1}{n}\right)$.
2. Show that intersection of two fields is a field.
3. Define quantile function. Obtain the quantile function of exponential random variable.
4. State Lindeberg- Feller central limit theorem.
5. Define expectation and list out its properties.
6. State dominated convergence theorem. Mention its application.
7. Prove that moment generating function (MGF) of sum of independent random variable is product of MGF of individual random variables.
8. State continuity theorem of characteristic function. Mention its application.

**PART B**

**Answer any FOUR of the following 13x 4=52**

1. A) Distinguish between field and $σ $field. (3)

B) With usual notations prove that $μ\left(\lim\_{n\to \infty }A\_{n}\right)=\lim\_{n\to }μ\left(A\_{n}\right)$ for a monotonic sequence $\left\{A\_{n}\right\}$ provided $μ\left(A\_{1}\right)<\infty $ when $\left\{A\_{n}\right\}$ is monotonically decreasing. (10)

1. A) Define Axiomatic approach of the probability. State and prove addition theorem of probability. (5)

B) State and prove Chebyshev’s inequality. Mention its application (5)

C) Prove that if A and B are independent then A and $B^{c}$ are also independent. (3)

1. A) If $F$ is a distribution function, then prove that$ $it satisfies following properties
2. $F\left(\infty \right)=1$ and $F\left(-\infty \right)=0$,
3. $F$ is right continuous and non-decreasing on **R.** (6)

 B) State and prove khintchine’s theorem. (7)

1. A) State and Prove Holder’s inequality. (8)

B) For a random variable X prove that $E\left(X^{s}\right)$ exists if $E\left(X^{r}\right)$ exists, where $ r>s$. (5)

1. A) Let $\left\{X\_{n}\right\}$ be a sequence of iid random variables with $E\left(X\_{i}\right)=0$ and $E\left(X\_{i}^{2}\right)=0$ for all $i$ and $cov\left(X\_{i}, X\_{j}\right)=\left\{\begin{array}{c}ρ if \left|i-j\right|=1\\0 otherwise\end{array}\right.$. Verify whether WLLN holds for $\left\{X\_{n}\right\}$ (7)

B) For a symmetric (about zero) random variable prove that characteristic function is even and real. (4)

C) Define convergence in probability. (2)

1. A) Derive the moment generating function of chi-square distribution. (5)

B) State and prove inversion theorem. (8)