ST. JOSEPH'S COLLEGE (AUTONOMOUS), BENGALURU-27
M.Sc. BIG DATA ANALYTICS - I SEMESTER
SEMESTER EXAMINATION: FEBRUARY 2022
(Examination conducted in January-March 2022)
BDA 1321 – LINEAR ALGEBRA AND LINEAR PROGRAMMING

TIME: 2.5 HOURS   **MAX MARKS: 70**

This question paper contains FOUR pages and THREE parts

Notations (unless stated otherwise):

α, β	scalar
r, u, v, w	vector
R	real line
$\in$	belongs to
$(\dots)$	anything enclosed within () is a column vector
$w.r.t.$	with respect to

PART-A

Answer ALL 20 questions (MCQs: only ONE correct answer)   **20 X 1 = 20**

1. Geometric Vectors in 3D space are directed line segments with *tail* at the ordered triplet $(1,1,1)$.
    a. True
    b. b. False
2. Which of the following is NOT true? $\alpha(u + v) =$
    a. $\alpha(v + u)$
    b. b. $\alpha u + v$
    c. c. $\alpha u + \alpha v$
3. $(1,0) \in \text{Span}(0,1)$.
    a. True
    b. b. False
4. Which of the statements is NOT true for two geometric vectors $u \,\&\, v$.
    a. Linearly Independent iff they do NOT lie on the same line
    b. Linearly Independent iff $\alpha u + \beta v = 0$ only when $\alpha = 1 \,\&\, \beta = 0$
    c. Linearly Independent iff $\alpha u + \beta v = 0$ only when $\alpha = 0 \,\&\, \beta = 0$
5. The set $\{(1,0), (0,0)\}$ is Linearly Independent.
    a. True
    b. b. False
6. The shape of each grid in Skewed Cartesian Grids is a Square
    a. True
    b. b. False
7. Which of the following function is Bijective?

a. $f(x) = x$
b. b. $f(x) = x^2$
c. c. $f(x) = |x|$

8. Which of the following is a valid decomposition of $r = (4\alpha, 3\alpha, 7\alpha) \ w.r.t. \ u = (4,0,0) \ \& \ v = (0,0,7)$?
Note: First coefficient scales u & second v.
   a. $4, \alpha$
   b.
   c. b. $\alpha, 7$
   d. c. α, α
   e. d. decomposition is NOT possible

9. $W = \{(\alpha, 0): \alpha \in R^+\}$ is a Vector Space.
   a. True
   b. b. False

10. $W = \{(\alpha, 0): \alpha \in R\}$ is a subspace of $V = \{(\alpha): \alpha \in R\}$.
   a. True
   b. b. False

11. Intersection of two lines in a plane is a Vector Space.
   a. True
   b. b. False

12. Given $B = \{(1,0), (0,1), (1,1)\} \ of \ R^2$. Which of the "criteria of a Basis set" does B violate?
   a. B is Linearly Independent
   b. b. $Span(B) = R^2$
   c. B is Linearly Dependent

13. What is the dimension of the Vector Space $W = \{(\alpha, \beta, \alpha + \beta): \alpha \in R, \beta \in R\}$?
   a. 1
   b. b. 2
   c. c. 3

14. In a matrix-vector multiplication, the elements of the vector scale the rows of the matrix.
   a. True
   b. b. False

15. Given a square matrix with linearly dependent columns, then determinant of the matrix is non-zero.
   a. True
   b. b. False

16. For which matrix given below, the determinant gets squished?
   a. $\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$
   b. $\begin{bmatrix} \pi & 0 \\ 0 & e \end{bmatrix}$
   c. $\begin{bmatrix} 0.25 & 0 \\ 0 & 0.75 \end{bmatrix}$

17. Given a square matrix with linearly independent columns, then which of the following is NOT true?
   a. The determinant is non-zero
   b. The matrix is a Bijective linear transformation
   c. The inverse of the matrix exists
   d. It has a non-trivial null space

18. What are the eigenvalues of reflection linear transformation?
   a. $1,0$
   b. b. $1, -1$

c. c. $0, -1$

19. Which of the following is NOT the null space of the matrix $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$?

   a. $\{\alpha(2, -1): \alpha \in R\}$
   b. b. $\{\alpha(2, -4): \alpha \in R\}$
   c. c. $\{\alpha(-8, 4): \alpha \in R\}$

20. Which is the following is the column space of the matrix $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$?

   a. R^1
   b. b. R^2
   c. c. R^3

PART-B

Answer ANY SIX questions   **6 X 5 = 30**

21. State any 5 properties of a vector.
22. Define Linearly Independent and Linearly Dependent sets. Illustrate with two examples for each. Explain the examples both in terms of algebra and geometry wherever applicable. (1+2+2)
23. Given three vectors in 3D space. What is(are) the span of these three vectors.
24. Define a Vector Space. Illustrate an example in Geometric Vector Space and give two spanning sets for the example chosen. Similarly, illustrate an example in R^n Vector Space and give two spanning sets for the example chosen.   (1+2+2)
25. Show that $A = \{\alpha(1,1): \alpha \in R\}$ & $B = \{\alpha(2,1): \alpha \in R\}$ are subspaces of R^2. Also, show that $A \cap B$ is a subspace of R^2.   (2+2+1)
26. Briefly explain the terms: Linear Programming, Objective Function, Constraints & Optimization Problems.
27. Solve the following Linear Programming Problem graphically:
$$maximize \quad z = 4x + y \quad w.r.t. constraints \quad x + y \leq 50, 3x + y \leq 90, x \geq 0, y \geq 0.$$
28. A manufacturing company makes two models A and B of a product. Each piece of Model A requires 9 labour hours for fabricating and 1 labour hour for finishing. Each piece of Model B requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available are 180 and 30 respectively. The company makes a profit of Rs. 8000 on each piece of model A and Rs. 12000 on each piece of Model B. How many pieces of Model A and Model B should be manufactured per week to realise a maximum profit? What is the maximum profit per week?

PART-C

Answer ANY TWO questions   **2 X 10 = 20**

29. Show that $W = \{(\alpha, \beta, \alpha + \beta, \alpha - \beta): \alpha \in R, \beta \in R\}$ is a vector space. Find a basis set and dimension of W.   (3+5+2)
30.
   a. Solve the following system of linear equations using Gaussian Elimination (Use column and null spaces).   (5)
$$2x + 2z = 0$$
$$2x + y + 2z = 1$$
$$w + 4x + 3y + 4z = 0$$

Note: w, x, y & z are variables, not vectors.

    b. When a matrix is said to be in Row Reduced Echelon Form (RREF)? What is the rank of a matrix? What is the role of RREF in calculating the rank? Illustrate with an example.

                  (1+2+2)

31. Explain eigenvalues and eigenvectors of

    a. any square matrix with linearly independent columns.   (3)

    b. any square matrix with linearly dependent columns.   (3)

    c. projection linear transformation.   (4)