Register Number:

Date:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27
M.Sc. MATHEMATICS - II SEMESTER
SEMESTER EXAMINATION: APRIL 2018

MT 8214: COMPLEX ANALYSIS

Time- 2 1/2 hrs **Max Marks-70**

This paper contains TWO printed pages

Answer any SEVEN questions from the following.

1. a) Evaluate $\oint_c \frac{dz}{(z-a)^n}$, $(n \neq 1)$ where $c: |z-a| = r$.
   b) State and Prove Morera's Theorem. **(5+5)**
2. a) Define analytic function and give an example.
   b) State and Prove Cauchy's Theorem for a triangle **(2+8)**
3. a) State and Prove Lioville's Theorem.
   b) Prove that "The cross ratio of a set of four points is preserved under a Bilinear Transformation". **(5+5)**

4. a) Define Zero of an analytic function.
   b) Find the radius of convergence of the power series $\sum \frac{z^n}{2^{n+1}}$.
   c) Let $\sum a_n(z-z_0)^n$ be a complex power series about the point z_0, then there exists a number 'R' is called the Radius of convergence of $\sum a_n(z-z_0)^n$, then Prove the following:
   (i) The power series $\sum a_n(z-z_0)^n$ converges absolutely for $|z-z_0| < R$.
   (ii) The power series $\sum a_n(z-z_0)^n$ converges uniformly for $|z-z_0| \leq r$,. where $0 < r < R$.
   (iii) If $|z-z_0| > R$. then the power series $\sum a_n(z-z_0)^n$ diverges. **(1+2+7)**
5. a) State and Prove Taylor's Theorem.
   b) Expand $\frac{z}{(z-1)(2-z)}$ in Laurentz series valid for $|z-1|$ < 1. **(5+5)**

6. a) State and Prove Cauchy Residue Theorem.

   b) Evaluate $\int_{\gamma} \frac{z^2 - z + 1}{(z-1)(z-4)(z+3)}$, $\gamma : |z| = 5$, using Cauchy Residue Theorem. **(5+5)**

7. a) Show that $\int_0^{\infty} \frac{dx}{x^4 + a^4} = \frac{\pi}{2\sqrt{2}\,a^3}$.

   b) Evaluate $\int_0^{\infty} \frac{\cos x}{x^2 + 1} dx$. **(6+4)**

8. a) State and Prove Rouche's Theorem.

   b) State and Prove Weierstrass factorization Theorem. **(7+3)**

9. State and Prove Phragmen- Lindelof Theorem. **(10)**

10. State and Prove Poisson- Jensen Formula. **(10)**