Register Number:
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ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27

M.Sc. Physics - III SEMESTER

SEMESTER EXAMINATION: JANUARY 2021

PH7220– MATHEMATICAL PHYSICS

Time- 2 1/2 hrs Max Marks-70

This paper contains THREE printed pages and TWO parts

Section – A

Answer any FIVE from the following questions. Each question carries 10 Marks.

(5 x 10 = 50)

1. A vector space is an abelian set $V$with extra structure with 2 defined operators, namely $+$, vector addition and $\odot$, scalar multiplication. This means that with the operators it satisfies the following conditions
Vectors Addition $+: V+V \to V$

   a) Commutative :- $v+u = u+v \, \forall (u, v) \in V$
   b) Associative :- $v+(u+w) = (v+u)+w$
   c) Identity :- $\exists 0 \in V: v+0 = v$
   d) Inverse :- $\exists u : u+v = 0$

   Scalar multiplication $\odot: \mathbb{R} \odot V \to V$

   a) Associative :- $\lambda(\gamma \odot v) = (\lambda\gamma) \odot v \, \forall (\lambda, \gamma) \in \mathbb{R}$
   b) Distributive :- $\lambda \odot (v+u) = \lambda \odot v + \lambda \odot u$
   c) Distributive :- $(\lambda+\gamma) \odot v = \lambda \odot v + \gamma \odot v$
   d) Identity :- $\exists I \in \mathbb{R}: I \odot v = v$

   Under these rules **take any one function of your choice** and show that it will also **satisfy all these conditions.**

2. Find the series solutions in descending power of x for Legendre's differential equation. Obtain Legendre function of 1st kind equation? Where n is positive integer.

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0$$

3. a). Find the first three terms of the Taylor series expansion of the complex variable function: $f(Z) = \frac{1}{(z^2+4)}$ about $z = -i$. Find the region of convergence.

   b). Using Cauchy's Residue theorem, determine the poles of the following function and residue at each pole: $f(z) = \int_c \frac{z^2}{(z-1)^2(z-2)} dZ$, where C is $|z| = 3$. **(5+5)**

4. Determine the solution of one dimensional heat equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ with the boundary conditions: $u(x,0) = 3sin\,(n\pi x),\ u(0,t),\ u(l,t) = 0$, where $0 < x < l$.

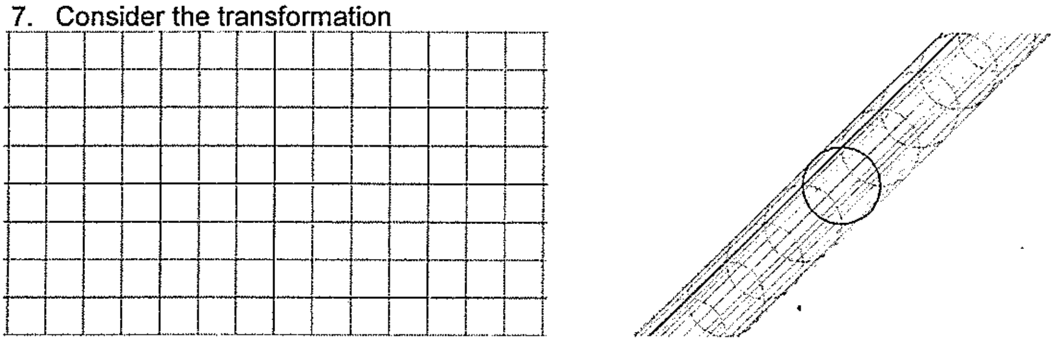
5. a).Evaluate the following complex integration using Cauchy's integral formula, $\int_C \frac{3z^2+z+1}{(z^2-1)(z-3)} dz$, Where C is the circle $|z| = 2$.

   b). Show that the function $u = \frac{1}{2}\log(x^2 + y^2)$ is harmonic. Find its harmonic conjugate.

   . **(5+5)**

6. A tensor is a mathematical quantity that undergoes the transformation $T'_{ij} = r_i^k r_j^l T_{kl}$ where r_p^o is a rotation matrix (any index) ,T_{kl} is the tensor in the unprimed frame and T'_{ij} is the tensor in the primed frame of reference. Then if $A_{ij}x^i x^j = C$ is the equation for an ellipsoidal surface centered at origin, then show that A_{ij} is a rank 2 tensor.

7. Consider the transformation



given by the transform laws

$$x' = \sin(x) + y$$
$$y' = \cos(x) + y$$

a) Express this transformation through a matrix. **[Show the working and the logic]**

b) Evaluate the Eigen values of the matrix at the point $x = \frac{\pi}{2}$ and $y = 3$. **(5+5)**

Section B

Answer any FOUR from the following questions. Each question carries 5 Marks.

[4 x 5 = 20]

8. Using Bessel's function, show that

   a) $J_{n+3} + J_{n+5} = \frac{2}{x}(n+4)J_{n+4}$.

   b) express $J_4(x)$ in terms of $J_0(x)$ and $J_1(x)$

   c) express $J_2(x)$ in term of $J_0(x)$ and $J_1(x)$.

(2+2+1)

9. Convert ordinary polynomial $64x^4 + 8x^3 - 32x^2 + 40x + 10$ into Hermite polynomial.

10. Evaluate the residues of the function $\frac{z^2}{(z-1)(z-2)(z-3)}$ at z = 1,2,3 and infinity. Show that their sum is zero.

11. Find the Laplace transform of the given functions: $(i)\ (1+\sin 2t),\ (ii) \cos\ at$.

(3+2)

12. Using Parseval's identity. Prove that $\int_0^{\infty}\left(\frac{sint}{t}\right)^2 dt = \frac{\pi}{2}$.

13. Determine whether the following equations are hyperbolic, parabolic and elliptic?

$(a).x^2\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = u, \qquad (b).t\frac{\partial^2 u}{\partial t^2} + 2\frac{\partial^2 u}{\partial x \partial t} + x\frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial x} = 0,$

$(c).\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}$

(2+2+1)

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