Register Number:
Date:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27
M.Sc. MATHEMATICS - IV SEMESTER
SEMESTER EXAMINATION: APRIL 2018
MT-0416 – THEORY OF NUMBERS

Time- 2 ½ hrs. **Max Marks-70**

This paper contains 1 printed page.

Answer any seven questions. **(7x10=70)**

1. a) If f and g are multiplicative, prove that their Dirichlet product is multiplicative.

   b) If g and $f*g$ are multiplicative, then prove that f is also multiplicative. (4+6)

2. a) Let f be multiplicative, then prove that f is completely multiplicative iff $f^{-1}(n)=\mu(n)f(n), \forall n\geq 1.$

   b) State and prove uniqueness theorem with respect to multiplicative functions. (7+3)

3. Write the partition for 6, 7, 8 and 9. (10)
4. Solve for x in $x\equiv 2(\text{mod}3), x\equiv 2(\text{mod}5), x\equiv 3(\text{mod}7), x\equiv 6(\text{mod}11)$ (10)
5. Evaluate $(-1\,|\,p)$ and $(2\,|\,p)$. (10)
6. State and prove Quadratic reciprocity law. (10)
7. Let p be an odd prime. Then prove the following,

   a) If g is a primitive root modulo p then g is also a primitive root modulo p^{α} for all $\alpha\geq 1$, if and only if g^{p-1} is not congruent to 1 modulo p^2.

   b) There is at least one primitive root $g \bmod p$ which satisfies g^{p-1} is not congruent to 1 modulo p^2, hence there exist at least one primitive root $\bmod p^{\alpha}$ if $\alpha\geq 2$. (10)

8. Prove that, for $|x|<1$, we have $\prod_{m=1}^{\infty}\frac{1}{1-x^m}=\sum_{n=0}^{\infty}p(n)x^n$, where $p(0)=1$. (10)
9. State and prove Euler's pentagon-number theorem. (10)
10. State and prove Jacobi's triple product identity. (10)