Register Number:

DATE:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27
M.Sc. MATHEMATICS – IV SEMESTER
SEMESTER EXAMINATION: APRIL 2018
MT 0214: GRAPH THEORY

Time- 2 ½ hrs **Max Marks-70**

This paper contains two printed pages.

Answer any seven questions. **(7x10=70)**

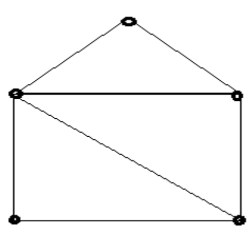
1. If G is a block prove that every two points of G lie on a common cycle where G is any connected (p,q) graph with $p \geq 3$. (10)

2. Prove that every planar (p,q) graph G, with $p \geq 4$ has at least 4 vertices of degree not exceeding 5. (10)

3. For any graph G, prove that the sum and product of χ and $\overline{\chi}$ satisfy the inequalities, $2\sqrt{p} \leq \chi + \overline{\chi} \leq p+1$ and $p \leq \chi\overline{\chi} \leq \left(\frac{p+1}{2}\right)^2$ where χ is the chromatic number of G and $\overline{\chi}$ is the chromatic number of $\overline{G}$. (10)

4. State and prove five color theorem. (10)

5. a) Find the chromatic polynomial of the following graph.



   b) Prove that a graph G with p vertices is a tree if and only if $f(G,t) = t(t-1)^{p-1}$ where $f(G,t)$ is the chromatic polynomial of G. (4+6)

6. State and prove Hall's theorem for bipartite graphs. (10)

7. State and prove Havel Hakimi theorem. (10)

8. Prove that a nontrivial connected digraph D is Eulerian if and only if $od(v) = id(v)$ for every vertex v of D. (10)

9. a) If u is a vertex of maximum out degree in a tournament T, then prove that $\vec{d}(u, v) \leq 2$ for every vertex v in T.

   b) Prove that every tournament contains a Hamiltonian path. (5+5)

10. a) Prove that $\gamma(C_n) = \left\lceil \frac{n}{3} \right\rceil$ for $n \geq 3$ where γ is the domination number.

    b) Prove that for every graph G containing no isolated vertices, $\gamma(G) \leq \gamma_t(G) \leq 2\gamma(G)$ where γ_t is the total domination number. (6+4)