Register Number:

DATE:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27
M.Sc. MATHEMATICS – IV SEMESTER
SEMESTER EXAMINATION: APRIL 2018
MT 0214: GRAPH THEORY

Time- 2 ½ hrs **Max Marks-70**

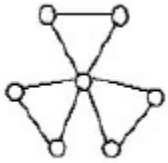
This paper contains two printed pages.

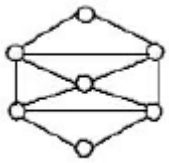
Answer any seven questions. **(7x10=70)**

1. State and prove Menger's Theorem. (10)

2. a) Define the crossing number of a graph. Draw a graph with crossing number one.
   b) Prove or disprove: K_5 and $K_{3,3}$ are planar.
   c) If G is a maximal outer planar graph with $p \geq 3$ vertices, all lying on the exterior face, then prove that G has $p-2$ interior faces. (2+4+4)

3. a) Give the chromatic number and edge chromatic number of the following graphs.





   b) For any (p,q) graph G, prove that $\frac{p}{\beta_0} \leq \chi(G) \leq p - \beta_0 + 1$ where β_0 is the point independence number and χ is the chromatic number of G. (4+6)

4. What is the four color conjecture? Prove that four color conjecture holds if and only if every cubic bridgeless plane map is four colorable. (10)

5. State and prove Konig's Theorem. (10)

6. Prove that a graph G is 2-factorable if and only if G is r-regular for some positive even integer r. (10)

7. Prove that a nontrivial connected graph G has a strong orientation if and only if G contains no bridge. (10)

8. Prove that a nontrivial tournament T is Hamiltonian if and only if T is strong. (10)

9. Define the edge independence number β_1 and edge covering number α_1 of a graph. For any nontrivial, connected (p,q) graph G, prove that $\alpha_1 + \beta_1 = p$ (10)

10. Define a minimal and minimum dominating set of a graph. If G is a graph with n vertices, then prove that $\frac{n}{1+\Delta(G)} \leq \gamma(G) \leq n - \Delta(G).$ (10)