Register Number:
Date:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27

B.Sc. MATHEMATICS – II SEMESTER

SEMESTER EXAMINATION: APRIL 2019

MT 218 MATHEMATICS

Time: 2 ½ hrs **Max Marks: 70**

This paper contains THREE printed pages and FOUR parts.

I Answer any FIVE of the following. **5 X 2 = 10**

1. In the group $(\mathbb{Z},*)$ of integers, where $a*b=a+b-1,\ \forall a,b\in\mathbb{Z}$, find the identity element and 2^{-1}.

2. In a group G, if $(ab)^2=a^2b^2,\ \forall a,b\in G,$ then prove that G is abelian.

3. If $f=\begin{pmatrix}1&2&3\\1&3&2\end{pmatrix}$ and $g=\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}$ be two elements of the symmetric group S_3 of three symbols $\{1, 2, 3\}$, find $g\,f\,g^{-1}$.

4. Negate: $(\forall x)[\sim p(x)\Rightarrow q(x)]$.

5. Find the angle between the radius vector and the tangent to the curve $r=a(1+\sin\theta)$ at any point (r,θ) on the curve.

6. Show that the polar sub tangent to the curve $r=3\sec 3\theta$ at any point on it, is $\operatorname{cosec} 3\theta$.

7. Find the volume of the solid generated by revolving the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ about the x-axis.

8. Find the integrating factor of the linear differential equation $(1+y^2)\,dx=(\tan^{-1}y-y)\,dy$.

II Answer any TWO of the following. **2 x 6 = 12**

9. Prove that the set of matrices $M = \left\{ \begin{bmatrix} x & x \\ x & x \end{bmatrix} : \ x \in \mathbb{R} \ and \ x \neq 0 \right\}$ forms an abelian group under multiplication of matrices.

10. (a) Prove that a non-empty sub set H of a group G is a sub group of G if and only if $ab^{-1} \in H, \ \forall \ a, b \in H.$ **[4]**

(b) Show that $SL_2(\mathbb{R}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : ad - bc = 1 \right\} \subseteq GL_2(\mathbb{R}),$ is a sub group of the group $\left(GL_2(\mathbb{R}), \cdot \right)$, the group of 2 x 2 non-singular real matrices under multiplication. **[2]**

11. If $p(x) \ and \ q(x)$ are open sentences with the same replacement set, then prove that $T[p(x) \Rightarrow q(x)] = \{T[p(x)]\}' \cup T[q(x)]$.

III Answer any FIVE of the following. **5 x 6 = 30**

12. With usual notations, for a curve $y = f(x),$ prove that $\frac{ds}{dx} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$. Hence deduce that $\frac{dx}{ds} = \cos\psi$ and $\frac{dy}{ds} = \sin\psi$.

13. Show that the pedal equation of the parabola $y^2 = 4a(x+a)$ is $p^2 = ar.$ Also show that the radius of curvature ρ at any point on the curve is proportional to p^3.

14. (a) Find the angle of intersection of the curves $r = \sin\theta + \cos\theta$ and $r = 2\sin\theta.$ **[4]**

(b) Find the nature of the double point (3, 2) on the curve $x^3 - 7x^2 - y^2 + 15x + 4y - 13 = 0$. **[2]**

15. Find all the asymptotes to the curve $x^3 + x^2y - xy^2 - y^3 + x^2 - y^2 - 2 = 0$.

16. Trace the curve Lemniscates of Bernoulli $r^2 = a^2\cos 2\theta.$

17. (a) Find the area of one loop of the three leaved rose $\mathrm{r} = \mathrm{a}\sin(3\theta)$. **[3]**

(b) Find the perimeter of the Cardioid $r = a(1-\cos\theta)$. **[3]**

18. Find the surface area of the solid generated by revolving the astroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ about the x-axis.

IV Answer any THREE of the following. 3 x 6 = 18

19. Solve : $x^2 \frac{dy}{dx} + xy = y^2 \log_e x$

20. Solve by finding a suitable integrating factor: $y(4x+y)dx - 2(x^2 - y)dy = 0$.

21. Find the general and the singular solutions of $x^2(y-px) = y\,p^2$ by using the substitutions $x^2 = u$ and $y^2 = v$.

22. Find the orthogonal trajectory of the family of conics $\frac{x^2}{a^2} + \frac{y^2}{a^2+\lambda} = 1$, where λ is a parameter.
