Register Number:

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ST. JOSEPH'S COLLEGE (AUTONOMOUS), BENGALURU-27

M.Sc. STATISTICS - I SEMESTER

SEMESTER EXAMINATION - DECEMBER 2020

STA7620 – LINEAR ALGEBRA

Time: 1½ hrs **Max: 35 Marks**

This question paper has **ONE** printed page and **TWO** parts

Use of scientific calculator is **NOT** allowed

SECTION – A

I Answer any THREE of the following: **3 x 3 = 9**

1. Define basis and dimension of vector space.
2. Define orthogonal vectors and orthogonal matrix. Provide an example for each.
3. If A is square matrix, show that A^l is idempotent matrix iff A is idempotent matrix.
4. Define characteristic roots and characteristic vectors. List out any two properties.
5. Write a note on singular value decomposition and its applications.

SECTION – B

II Answer any TWO of the following: **2 x 13 = 26**

6. A) Outline the procedure of finding determinant of a square matrix by partitioned method. (4)

   B) Define trace of a square matrix and give any two properties. (2)

   C) Setup a linear transformation Y=Ax which carries E_1 to $Y_1= [1,2,3]^l$, E_2 to $Y_2= [3,1,2]^l$ and E_3 to $Y_3= [2, -1, -1]^l$ (7)

   i) Show that the transformation is singular

   ii) Find images of $X_1 = [1, 1, 1]^l$, $X_2 = [2, 0, 2]^l$ and $X_3 = [3, 4, 1]^l$

   iii) Examine linear independency of images of X_1, X_2 and X_3 obtained in ii)

7. A) Construct orthonormal basis of $V_3(R)$, using Gram-Schmidt process, given the vectors

   $x_1= (1, 2, 0)^l$, $x_2= (8, 1, -6)^l$, $x_3=(0, 0, 1)^l$ (6)

   B) State and prove Cayley-Hamilton theorem (7)

8. A) Examine consistency of the following system of linear equations (6)

   $2x + 6y + 11 = 0$

   $x + 20y - 6z + 3 = 0$

   $6y - 18z + 1 = 0$

   B) Explain need for generalized inverse and define generalized inverse. Write down Moore-Penrose matrix equations with reference to above system of linear equations (8. A) (7)