Register Number:
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ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27

M.Sc. PHYSICS - I SEMESTER

SEMESTER EXAMINATION- JANUARY 2021

PH 7520 – ANALYTICAL TOOLS FOR MATHEMATICAL PHYSICS

Time-1 hrs. **Maximum Marks-30**

This question paper has 2 printed pages and 1 part

PART A
Answer any THREE full questions

1. Consider the following vectors $|v\rangle=\begin{pmatrix}1\\2\\5\end{pmatrix}$ and $|u\rangle=\begin{pmatrix}3\\0\\-3\end{pmatrix}$

   a) Compute $\||v\rangle\|$ and $\||u\rangle\|$
   b) Compute the inner product of the vectors
   c) What is the angle between these vectors?
   d) Consider a vector $|w\rangle=\begin{pmatrix}1\\x\\y\end{pmatrix}$. Compute the values of x and y such that all 3 vectors $(|v\rangle;|u\rangle \text{and} |w\rangle)$ are mutually perpendicular

(2,2,2,4)

2. A mobius strip is defined by the function $\log(r)\sin(\theta)=z\cos(\theta)$
   a) Which coordinate frame is the mobius strip defined in?
   b) If the transformation from Cartesian to cylindrical coordinates are given by

   $$x=r\sin\theta$$
   $$y=r\cos\theta$$
   $$z=z$$

   rewrite the ∇ operator in cylindrical coordinates

   c) Compute the normals to this surface

(1,6,3)

3.

a) Are the vectors $|a\rangle=\begin{pmatrix}1\\2\\3\end{pmatrix}$, $|b\rangle=\begin{pmatrix}3\\1\\7\end{pmatrix}$ and $|c\rangle=\begin{pmatrix}5\\5\\13\end{pmatrix}$ linearly independent? Show it by row reduction

b) Orthogonalise $|v\rangle=\begin{pmatrix}1\\0\\3\end{pmatrix}$, $|b\rangle=\begin{pmatrix}3\\1\\7\end{pmatrix}$ and $|w\rangle=\begin{pmatrix}2\\1\\3\end{pmatrix}$ using the Gram Schmidt process

(5,5)

4. Consider the vector $|p\rangle=\begin{pmatrix}4\\6\\1\end{pmatrix}$

a) Decompose the vector into its basis space components. [$|u\rangle=a_i B_i$ **where** $a_i\in\mathbb{R}$ **is the** i^{th} **component and** B_i **is the** i^{th} **basis vector]**

b) Write the transformation matrix for a rotation by $\theta=30^o$ about the x-axis. **[x-axis is kept fixed and serves as the axis of rotation]**

c) How will $|p\rangle$ look if it undergoes the transformation as defined by the matrix computed from above

(3,2,5)