Register Number:
DATE:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27
M.Sc. PHYSICS – II SEMESTER
SEMESTER EXAMINATION – APRIL 2019
PH 8118 : ELECTRODYNAMICS

Time: 2.5 hours **Maximum Marks:70**

This question paper contains 2 parts and 3 printed pages.

Some useful Identities:

$$\nabla(\vec{A}.\vec{B})=\vec{A}\,X(\nabla X\vec{B})+\vec{B}\,X(\nabla X\vec{A})+(\vec{A}.\nabla)\vec{B}+(\vec{B}.\nabla)\vec{A}$$

$$\vec{\nabla}.(f\vec{A})=f(\vec{\nabla}.\vec{A})=\vec{A}.(\vec{\nabla}f)$$

Part-A

Answer any 5 questions. Each question carries 10 marks. (10x5=50)

1. a) Explain why the work done in moving a charge on an equipotential surface is zero.
   b) The electric field of an oscillating dipole oscillating with frequency ω at distance $\vec{r}$ from the centre of it is given as : $\vec{E}(\vec{r},t)=\frac{-\mu_o p_o\omega^2}{4\pi}\frac{\sin\theta}{r}\cos[\omega(t-r/c)]\hat{\theta}$ and the magnetic field is given as : $\vec{B}(\vec{r},t)=\frac{-\mu_o p_o\omega^2}{4\pi c}\frac{\sin\theta}{r}\cos[\omega(t-r/c)]\hat{\phi}$ .Find the intensity of energy radiated by this oscillating dipole and explain the intensity profile. (2+8)

2. In general the vector potential of an arbitrary localized current distribution at a point at distance $\vec{r}$ from origin can be expanded in power series. This vector potential for a current loop is given as : $\boldsymbol{A}(r)=\frac{\mu_o I}{4\pi}\frac{\oint dl'}{\gamma}$ where dl' is the elemental length of this current loop whose distance from origin is $\vec{r}\,'$ and distance from the far-off point where potential is being determined is $\vec{\gamma}$. Using appropriate form of the above relation, find the divergence of this magnetic vector potential. Also, state its value in magnetostatics.

3. Write Maxwell's equations. Explain the basis for inclusion of Displacement current term in the fourth equation(corrected form of Ampere’s law) and it’s significance. With mathematical formulation show that even though the actual current enclosed between the two parallel plates of a charging capacitor is zero, there exists a current term which is equivalent to the actual current.

4. Calculate the total electromagnetic force on the charges in a given volume ‘V’ and show that the force per unit volume when expressed in terms of fields $\vec{E},\vec{B}$ can be written in terms of Maxwell’s stress tensor $\boldsymbol{T}$ and the Poynting vector $\vec{S}$ where $\nabla.\boldsymbol{T}=\epsilon_o[(\nabla.\vec{E})\vec{E}+(\vec{E}.\nabla)\vec{E}]+\frac{1}{\mu_o}[(\nabla.\vec{B})\vec{B}+(\vec{B}.\nabla)\vec{B}]-\frac{1}{2}\nabla(\epsilon_o E^2+\frac{1}{\mu_o}B^2)$. Also, find the total force and hence derive the momentum conservation relation for electromagnetic fields.

5. Starting with the general form of Maxwell's equations, obtain the potential formulation of these equations in terms of scalar and vector potentials $V, \vec{A}$. Stating the advantages of this formulation explain what are gauge transformations.

6. Suppose an x-y plane forms the boundary between two linear dielectric media at z=0. An incoming monochromatic plane wave of frequency 'ω', travelling in z-direction, polarized in the plane of incidence (x-z plane) meets the boundary at an angle θ_I. It gives rise to reflected wave at angle θ_R and transmitted wave at angle θ_T where $\theta_T < \theta_I$ as velocity of wave in medium 2 v_2 is less than than in medium 1 v_1 . Assume that all the three laws of geometrical optics are obeyed. Show that the Fresnel's equations for this polarization state are $\tilde{E}_{0R} = (\frac{\alpha-\beta}{\alpha+\beta})\tilde{E}_{0I}$ and $\tilde{E}_{0T} = (\frac{2}{\alpha+\beta})\tilde{E}_{0I}$ where $\tilde{E}_{0I}, \tilde{E}_{0R}, \tilde{E}_{0T}$ are the incident, reflected and transmitted amplitudes. Here $\alpha = \frac{\cos\theta_T}{\cos\theta_I}$ And $\beta = \frac{\mu_1 v_1}{\mu_2 v_2}$ Assume $\mu_1 \approx \mu_2 \approx \mu_0$. Explain what is Brewster's angle?

7. Suppose you had a string of +ve charges moving along to the right at speed 'v' and superimposed on this +ve string is a -ve string proceeding to the left with the same speed 'v' such that I=2λv in reference frame S.(Assume that the charges are close enough so that linear charge density is λ. Also, the line charge density in the rest frame of charges is λ_o). Now, suppose there is a charge +q at a vertical distance 's' away from this assembly which moves with speed 'u' such that u<v. The net electrical force on this charge in this frame S is zero.
   Now, analyse the system from another frame $\bar{S}$ which is moving to the right at a speed 'u' and find the force experienced by this charge in this frame $\bar{S}$. Using the transformation rules for forces i.e. $\bar{F}_{\perp} = \frac{F_{\perp}}{\gamma}$ where $F_{\perp}$ is the force in the frame in which the speed of particle is zero and $\gamma = \frac{1}{\sqrt{1-u^2/c^2}}$, find the force experienced by the charge q in the frame S. Discuss what is the nature of the force in frame S.

Part-B

Answer any 4 questions. Each question carries 5 marks. (4x5=20)

8. Consider an axially symmetric static charge distribution of the form $\rho = \rho_o (\frac{r_o}{r'})^2 e^{(-r'/r_o)} \cos^2\phi$. Find the radial component of the dipole moment due to this entire charge distribution in terms of ρ_o, r_o .

9. Consider an infinite conducting sheet in the xy -plane with a time dependent current density $Kt\hat{i}$, where K is a constant. The vector potential at a point P(x,y,z) is given by $\vec{A} = \frac{\mu_o K}{4c}(ct-z)^2\hat{i}$. Find the magnetic field at this point.

10. The electric and magnetic fields that propagate as plane waves in conductors are of the form $\vec{E}(z,t) = \tilde{E}_o e^{i(\tilde{k}z-\omega t)}$ and $\vec{B}(z,t) = \tilde{B}_o e^{i(\tilde{k}z-\omega t)}$ where propagation vector $\tilde{k}$ is a complex quantity. Here $\tilde{k} = k + i\kappa$ .The real and imaginary parts are given as $k = \omega\sqrt{\frac{\epsilon\mu}{2}}[\sqrt{[1+(\frac{\sigma}{\epsilon\omega})^2]}+1]^{(1/2)}$ and $\kappa = \omega\sqrt{\frac{\epsilon\mu}{2}}[\sqrt{[1+(\frac{\sigma}{\epsilon\omega})^2]}-1]^{(1/2)}$. Find the skin depth for good and poor conductors.

11. The average lifetime of a π meson in its own frame of reference is 26.0 ns. (This is its proper lifetime.) If the π meson moves with speed 0.95c with respect to lab frame, what is its lifetime as measured by an observer at rest in this lab frame? What is the average distance it travels before decaying as measured by an observer at rest in the lab frame?

12. The scalar and vector potential corresponding to a charge and current distribution is given as $V(\vec{r},t)=0$ $\vec{A}(\vec{r},t)=\frac{-1}{4\pi\epsilon_o}\frac{qt}{r^2}\hat{r}$ where symbols have their usual meaning. Do these potentials obey Coulomb or Lorentz gauge?

13. Show that the scalar product of two four- dimensional vectors is invariant under Lorentz transformation where the transformation matrix M for transforming from reference frame S to $\bar{S}$ is given as $M=\begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$. Here $\gamma=\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$, $\beta=\frac{v}{c}$ and v is the velocity of $\bar{S}$ relative to S .