Register Number:
DATE:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27

B.Sc. MATHEMATICS– I SEMESTER

SEMESTER EXAMINATION: OCTOBER 2019

MT 118 : MATHEMATICS PAPER I

Time- 2 ½ hrs **Max Marks-70**

This question paper contains **FOUR** parts and **TWO** printed pages.

I. Answer any FIVE of the following. (5 X 2 = 10)

1. Find the rank of the matrix $A = \begin{pmatrix} 1 & -7 & 15 & -14 \\ 2 & 3 & -4 & 6 \\ 3 & -4 & 11 & -8 \\ 5 & -1 & 7 & -2 \end{pmatrix}$.

2. For what values of λ and μ the following system has an infinite number of solution $x + y + z = 6$; $x + 2y + 3z = 10$; $x + 2y + \lambda z = \mu$. Justify.

3. Find the n^{th} derivative of $\cos(ax + b)$.

4. If $x = r\cos\theta\ , y = r\sin\theta\ then\ find\ \frac{\partial(r,\theta)}{\partial(x,y)}$.

5. Evaluate $\int_0^{\frac{\pi}{4}} tan^5 x\, dx$.

6. Show that the planes $2x - 4y + 3z + 5 = 0\ and\ 10x + 11y + 8z - 17 = 0$ are perpendicular.

7. Find the angle between the line $\frac{x-3}{2} = \frac{y-1}{1} = \frac{z+4}{-2}$ and the plane $x + y + 4 = 0$.

8. Find the equation of the sphere which passes through $(-1,2,3)$ and has its centre at $(3,-1,1)$.

II. **Answer any THREE of the following.** **(3X6=18)**

9. Reduce the following matrix to its normal form and hence find its rank

$$A = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & -1 & 2 & 5 \\ 3 & 1 & 1 & 8 \\ 2 & -2 & 3 & 7 \end{bmatrix}.$$

10. Find the inverse of the matrix $A = \begin{bmatrix} 1 & -3 & 2 \\ -3 & 3 & -1 \\ 2 & -1 & 0 \end{bmatrix}$ by elementary operations.

11. Test the consistency and solve:

$x + 2y - 5z = -13\,; 3x - y + 2z = 1; 2x - 2y + 3z = 2$ and $x - y + z = -1$.

12. Diagonalise the matrix $A = \begin{bmatrix} 2 & 4 \\ 0 & 5 \end{bmatrix}$.

III. **Answer any FIVE of the following.** **(5X6=30)**

13. If $y = \left(x + \sqrt{x^2 - 1}\right)^m$ show that $(x^2 - 1)y_{n+2} + (2n + 1)xy_{n+1} + (n^2 - m^2)y_n = 0$.

14. State and prove Euler's theorem and its extension for homogeneous functions.

15. If $u = f(r) where\ r = \sqrt{x^2 + y^2}$, show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$.

16. (i) If $z = e^{ax+by} f(ax - by)$ show that $b\frac{\partial z}{\partial x} + a\frac{\partial z}{\partial y} = 2abz$

(ii) If $z = \tan^{-1}\left(\frac{y}{x}\right)$ where $y = tan^2 x$ find $\frac{dz}{dx}$ (4+2)

17. If $u = \frac{yz}{x}\,, v = \frac{zx}{y}\,, w = \frac{xy}{z}$ show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = 4$

18. (i) Write the reduction formula for $\int_0^{\frac{\pi}{2}} sin^m x cos^n x dx$

(ii) Evaluate $\int_0^{\pi} x\ sin^4 x cos^6 x dx$ (2+4)

19. Evaluate $\int_0^{\infty} \frac{\tan^{-1} ax}{x(1+x^2)}\, dx$, where a is a parameter, by applying differentiation under integral sign.

IV. **Answer any TWO of the following.** **(2X6=12)**

20. Find the equation of the plane containing the line $\frac{x+2}{1} = \frac{y-3}{2} = \frac{z+4}{-3}$ and passing through the point $(1,3,2)$

21. Find the shortest distance between the lines $\frac{x-3}{1} = \frac{y-4}{-2} = \frac{z+2}{-1}$ and $3x - y - 10 = 0 = 2x - z - 4$

22. Find the equation of the sphere which touches the plane $3x + 2y - z + 2 = 0$ at $(1, -2, 1)$ and passing through the origin.