Register Number:

Date:

ST JOSEPH'S UNIVERSITY, BENGALURU-27
B.Sc. (MATHEMATICS) - I SEMESTER
SEMESTER EXAMINATION: OCTOBER 2023
(Examination conducted in November/ December 2023)
MTOE 2: MATHEMATICS FOR PHYSICAL SCIENES- I
(For current batch students only)

Time: 2 Hours **Max Marks:** 60

This question paper contains **TWO** printed pages and **THREE** parts.
Normal **calculator** is allowed to use.

PART A

ANSWER ANY SIX OF THE FOLLOWING. (6×2=12)

1. Define rank of a matrix.
2. Find the value(s) of h such that the matrix is the augumented matrix of a consistent linear system. $A = \begin{bmatrix} 2 & 3 & h \\ 4 & 6 & 7 \end{bmatrix}$
3. Is $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$ an eigenvector of $A = \begin{bmatrix} -3 & 1 \\ -3 & 8 \end{bmatrix}$? Why or why not?
4. For what value of h and k the following system has no solution

$$\begin{aligned} x_1 + hx_2 &= 2 \\ 4x_1 + 8x_2 &= k. \end{aligned}$$

5. Evaluate $\lim_{x \to 2} \sqrt{4x^2 - 3}$.
6. State Lagrange's Mean Value Theorem.
7. Find the derivative of the function $x^2 cosx$.
8. Evaluate $\int_0^{\frac{\pi}{2}} sin^5 x \, dx$.

PART B

ANSWER ANY TWO OF THE FOLLOWING. (2×6=12)

9. Find the value of 'a' for which the following matrix has rank 3:

$$A = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 2 & -8 \\ -4 & 5 & a \end{bmatrix}.$$

10. Solve the following system of linear equations:

$$\begin{aligned} x+y+z &= 1 \\ x+2y+3z &= 4 \\ x+3y+5z &= 7 \\ x+4y+7z &= 10. \end{aligned}$$

11. Find the eigenvalues and the corresponding eigenvectors of the matrix $A = \begin{bmatrix} 5 & 3 \\ 3 & 5 \end{bmatrix}$.

PART C

ANSWER ANY <u>SIX</u> OF THE FOLLOWING. **(6×6=36)**

12. Verify Cayley Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}$.

13. Is $f(x) = x^2 - x + 3$ continuous at $x = 3$? Justify your answer using the definition of continuity.

14. Verify Cauchy mean value theorem for the function $f(x) = x^2 + 3, g(x) = x^3 + 1$ in $[1, 3]$.

15. Obtain the Maclaurin's series expansion of $e^x\ cosx$ upto the term containing x^3.

16. Evaluate the following limits using L'Hospital's Rule:

    i) $\lim_{x\to 0} \frac{e^x - e^{-x}}{sin(x)}$,

    ii) $\lim_{y\to 0} \left(\frac{1}{y} - \frac{1}{sin(y)}\right)$. (2+4)

17. Find the area bounded by the cardiod $r = a(1 + cos\ \theta)$.

18. Find the area of the surface generated by revolving the curve $x = a\ cos^3\theta,\ y = a\ sin^3\theta$ about the x-axis.

19. Find the volume of the solid genertaed by revolving the asteroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ about the x-axis.
