Registration Number:

Date & Session:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BENGALURU -27
M.Sc. MATHEMATICS – III SEMESTER
SEMESTER EXAMINATION: OCTOBER 2022
(Examination conducted in December 2022)
MT9622 : MATHEMATICAL METHODS

Time: 2 ½ Hours **Max Marks: 70**

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1. **The paper contains two pages.**
2. **Answer any SEVEN FULL questions.**
3. **Each question carries 10 marks.**

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1. Solve $\phi(x) = x - \int_0^x (x-t)\phi(t)\,dt$ by using Picard's method. Choose $\phi_0(x) = 0$ and perform three iterations for solution. [10M]
2. Reduce the initial value problem $\frac{d^3y}{dx^3} - 2xy = 0,\ y(0) = \frac{1}{2},\ y'(0) = 1 = y''(0)$ into an integral equation. [10M]
3. a) Evaluate $\int_0^1 (y'^2 - 2y - 2xy)\,dx,\ \ y(0) = 2,\ y(1) = 1$ by Rayleigh-Ritz's method. [5M]

   b) Solve the integral equation $\int_0^\infty f(\theta)\cos\alpha\theta\,d\theta = \begin{cases} 1-\alpha & 0 \le \alpha \le 1 \\ 0 & \alpha > 1 \end{cases}$

   using Fourier transform method and hence evaluate $\int_0^\infty \frac{\sin^2 t}{t^2}dt.$ [5M]
4. Find the complex Fourier transform of $e^{-a^2x^2},\ a > 0$. Hence deduce $e^{-\frac{x^2}{2}}$ is self reciprocal in respect of the complex Fourier Transform. [10M]
5. Find the Fourier sine and cosine transform of $f(x) = \begin{cases} x & 0 < x < 2 \\ 0 & otherwise \end{cases}$. [10M]
6. Find the asymptotic series of $\int_0^x t^{\frac{-1}{2}} e^{-t}dt$ as $x \to \infty$. [10M]

7. Using Watson's lemma evaluate $I(x)=\int_1^{\infty}(s^2-1)^{\frac{-1}{2}}e^{-xs}ds \quad as\ x\to\infty.$ [10M]

8. a) Find the leading ordered term of $\int_0^{\frac{\pi}{2}} e^{ix\cos t}dt \quad as \quad x\to\infty$. [5M]

b) Given $x^2+2x\varepsilon-3=0$, Obtain the power series expansion in ε using perturbation technique. [5M]

9. For small ε determine the first three terms in the expansion of roots of the equation

$x^2-(3+2\varepsilon)x+2+\varepsilon=0.$ [10M]

10. Solve the differential equation $\frac{dy}{dx}+y\varepsilon=x$, $y(0)=0$ using perturbation method. [10M]