Test Paper : **III**

Test Subject : **MATHEMATICAL SCIENCE**

Test Subject Code : **K-2616**

Test Booklet Serial No. : ____________

OMR Sheet No. : ____________

Roll No. | | | | | | | | |

(Figures as per admission card)

**Name & Signature of Invigilator/s**

Signature : ____________

Name : ____________

**Paper : III**

**Subject : MATHEMATICAL SCIENCE**

Time : 2 Hours 30 Minutes — Maximum Marks : 150

Number of Pages in this Booklet : **16** — Number of Questions in this Booklet : **75**

**ಅಭ್ಯರ್ಥಿಗಳಿಗೆ ಸೂಚನೆಗಳು**

1. ಈ ಪುಟದ ಮೇಲ್ತುದಿಯಲ್ಲಿ ಒದಗಿಸಿದ ಸ್ಥಳದಲ್ಲಿ ನಿಮ್ಮ ರೋಲ್ ನಂಬರನ್ನು ಬರೆಯಿರಿ.
2. ಈ ಪತ್ರಿಕೆಯು ಬಹು ಆಯ್ಕೆ ವಿಧದ ಎಪ್ಪತ್ತೈದು ಪ್ರಶ್ನೆಗಳನ್ನು ಒಳಗೊಂಡಿದೆ.
3. ಪರೀಕ್ಷೆಯ ಪ್ರಾರಂಭದಲ್ಲಿ, ಪ್ರಶ್ನೆಪುಸ್ತಿಕೆಯನ್ನು ನಿಮಗೆ ನೀಡಲಾಗುವುದು. ಮೊದಲ 5 ನಿಮಿಷಗಳಲ್ಲಿ ನೀವು ಪುಸ್ತಿಕೆಯನ್ನು ತೆರೆಯಲು ಮತ್ತು ಕೆಳಗಿನಂತೆ ಕಡ್ಡಾಯವಾಗಿ ಪರೀಕ್ಷಿಸಲು ಕೋರಲಾಗಿದೆ.
   (i) ಪ್ರಶ್ನೆ ಪುಸ್ತಿಕೆಗೆ ಪ್ರವೇಶಾವಕಾಶ ಪಡೆಯಲು, ಈ ಹೊದಿಕೆ ಪುಟದ ಅಂಚಿನ ಮೇಲಿರುವ ಪೇಪರ್ ಸೀಲನ್ನು ಹರಿಯಿರಿ. ಸ್ಟಿಕ್ಕರ್ ಸೀಲ್ ಇಲ್ಲದ ಅಥವಾ ತೆರೆದ ಪುಸ್ತಿಕೆಯನ್ನು ಸ್ವೀಕರಿಸಬೇಡಿ.
   (ii) **ಪುಸ್ತಿಕೆಯಲ್ಲಿನ ಪ್ರಶ್ನೆಗಳ ಸಂಖ್ಯೆ ಮತ್ತು ಪುಟಗಳ ಸಂಖ್ಯೆಯನ್ನು ಮುಖಪುಟದ ಮೇಲೆ ಮುದ್ರಿಸಿದ ಮಾಹಿತಿಯೊಂದಿಗೆ ತಾಳೆ ನೋಡಿರಿ. ಪುಟಗಳು/ಪ್ರಶ್ನೆಗಳು ಕಾಣೆಯಾದ, ಅಥವಾ ದ್ವಿಪ್ರತಿ ಅಥವಾ ಅನುಕ್ರಮವಾಗಿಲ್ಲದ ಅಥವಾ ಇತರ ಯಾವುದೇ ವ್ಯತ್ಯಾಸದ ದೋಷಪೂರಿತ ಪುಸ್ತಿಕೆಯನ್ನು ಕೂಡಲೆ 5 ನಿಮಿಷದ ಅವಧಿ ಒಳಗೆ, ಸಂವೀಕ್ಷಕರಿಂದ ಸರಿ ಇರುವ ಪುಸ್ತಿಕೆಗೆ ಬದಲಾಯಿಸಿಕೊಳ್ಳಬೇಕು. ಆ ಬಳಿಕ ಪ್ರಶ್ನೆ ಪತ್ರಿಕೆಯನ್ನು ಬದಲಾಯಿಸಲಾಗುವುದಿಲ್ಲ, ಯಾವುದೇ ಹೆಚ್ಚು ಸಮಯವನ್ನೂ ಕೊಡಲಾಗುವುದಿಲ್ಲ.**
4. ಪ್ರತಿಯೊಂದು ಪ್ರಶ್ನೆಗೂ (A), (B), (C) ಮತ್ತು (D) ಎಂದು ಗುರುತಿಸಿದ ನಾಲ್ಕು ಪರ್ಯಾಯ ಉತ್ತರಗಳಿವೆ. ನೀವು ಪ್ರಶ್ನೆಯ ಎದುರು ಸರಿಯಾದ ಉತ್ತರದ ಮೇಲೆ, ಕೆಳಗೆ ಕಾಣಿಸಿದಂತೆ ಅಂಡಾಕೃತಿಯನ್ನು ಕಪ್ಪಾಗಿಸಬೇಕು.
   **ಉದಾಹರಣೆ:** (A) (B) ● (D)
   (C) ಸರಿಯಾದ ಉತ್ತರವಾಗಿದ್ದಾಗ.
5. ಪ್ರಶ್ನೆಗಳಿಗೆ ಉತ್ತರಗಳನ್ನು, **ಪತ್ರಿಕೆ III ಪುಸ್ತಿಕೆಯೊಳಗೆ ಕೊಟ್ಟಿರುವ OMR ಉತ್ತರ ಹಾಳೆಯಲ್ಲಿ ಮಾತ್ರವೇ ಸೂಚಿಸತಕ್ಕದ್ದು.** OMR ಹಾಳೆಯಲ್ಲಿನ ಅಂಡಾಕೃತಿ ಹೊರತುಪಡಿಸಿ ಬೇರೆ ಯಾವುದೇ ಸ್ಥಳದಲ್ಲಿ ಗುರುತಿಸಿದರೆ, ಅದರ ಮೌಲ್ಯಮಾಪನ ಮಾಡಲಾಗುವುದಿಲ್ಲ.
6. OMR ಉತ್ತರ ಹಾಳೆಯಲ್ಲಿ ಕೊಟ್ಟ ಸೂಚನೆಗಳನ್ನು ಜಾಗರೂಕತೆಯಿಂದ ಓದಿರಿ.
7. ಎಲ್ಲಾ ಕರಡು ಕೆಲಸವನ್ನು ಪುಸ್ತಿಕೆಯ ಕೊನೆಯಲ್ಲಿ ಮಾಡತಕ್ಕದ್ದು.
8. ನಿಮ್ಮ ಗುರುತನ್ನು ಬಹಿರಂಗಪಡಿಸಬಹುದಾದ ನಿಮ್ಮ ಹೆಸರು ಅಥವಾ ಯಾವುದೇ ಚಿಹ್ನೆಯನ್ನು, ಸಂಗತವಾದ ಸ್ಥಳ ಹೊರತು ಪಡಿಸಿ, OMR ಉತ್ತರ ಹಾಳೆಯ ಯಾವುದೇ ಭಾಗದಲ್ಲಿ ಬರೆದರೆ, ನೀವು ಅನರ್ಹತೆಗೆ ಬಾಧ್ಯರಾಗಿರುತ್ತೀರಿ.
9. ಪರೀಕ್ಷೆಯು ಮುಗಿದನಂತರ, ಕಡ್ಡಾಯವಾಗಿ OMR ಉತ್ತರ ಹಾಳೆಯನ್ನು ಸಂವೀಕ್ಷಕರಿಗೆ ನೀವು ಹಿಂತಿರುಗಿಸಬೇಕು ಮತ್ತು ಪರೀಕ್ಷಾ ಕೊಠಡಿಯ ಹೊರಗೆ OMR ನ್ನು ನಿಮ್ಮೊಂದಿಗೆ ಕೊಂಡೊಯ್ಯಕೂಡದು.
10. ಪರೀಕ್ಷೆಯ ನಂತರ, ಪರೀಕ್ಷಾ ಪ್ರಶ್ನೆ ಪತ್ರಿಕೆಯನ್ನು ಮತ್ತು ನಕಲು OMR ಉತ್ತರ ಹಾಳೆಯನ್ನು ನಿಮ್ಮೊಂದಿಗೆ ತೆಗೆದುಕೊಂಡು ಹೋಗಬಹುದು.
11. **ನೀಲಿ/ಕಪ್ಪು ಬಾಲ್‌ಪಾಯಿಂಟ್ ಪೆನ್ ಮಾತ್ರವೇ ಉಪಯೋಗಿಸಿರಿ.**
12. **ಕ್ಯಾಲ್ಕುಲೇಟರ್, ವಿದ್ಯುನ್ಮಾನ ಉಪಕರಣ ಅಥವಾ ಲಾಗ್ ಟೇಬಲ್ ಇತ್ಯಾದಿಯ ಉಪಯೋಗವನ್ನು ನಿಷೇಧಿಸಲಾಗಿದೆ.**
13. **ಸರಿ ಅಲ್ಲದ ಉತ್ತರಗಳಿಗೆ ಋಣ ಅಂಕ ಇರುವುದಿಲ್ಲ.**
14. **ಕನ್ನಡ ಮತ್ತು ಇಂಗ್ಲೀಷ್ ಆವೃತ್ತಿಗಳ ಪ್ರಶ್ನೆ ಪತ್ರಿಕೆಗಳಲ್ಲಿ ಯಾವುದೇ ರೀತಿಯ ವ್ಯತ್ಯಾಸಗಳು ಕಂಡುಬಂದಲ್ಲಿ, ಇಂಗ್ಲೀಷ್ ಆವೃತ್ತಿಗಳಲ್ಲಿರುವುದೇ ಅಂತಿಮವೆಂದು ಪರಿಗಣಿಸಬೇಕು.**

**Instructions for the Candidates**

1. Write your roll number in the space provided on the top of this page.
2. This paper consists of seventy five multiple-choice type of questions.
3. At the commencement of examination, the question booklet will be given to you. In the first 5 minutes, you are requested to open the booklet and compulsorily examine it as below :
   (i) To have access to the Question Booklet, tear off the paper seal on the edge of the cover page. Do not accept a booklet without sticker seal or open booklet.
   (ii) **Tally the number of pages and number of questions in the booklet with the information printed on the cover page. Faulty booklets due to pages/questions missing or duplicate or not in serial order or any other discrepancy should be got replaced immediately by a correct booklet from the invigilator within the period of 5 minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given.**
4. Each item has four alternative responses marked (A), (B), (C) and (D). You have to darken the circle as indicated below on the correct response against each item.
   **Example** : (A) (B) ● (D)
   where (C) is the correct response.
5. Your responses to the question of Paper III are to be indicated in the **OMR Sheet kept inside the Booklet**. If you mark at any place other than in the circles in OMR Sheet, it will not be evaluated.
6. Read the instructions given in OMR carefully.
7. Rough Work is to be done in the end of this booklet.
8. If you write your name or put any mark on any part of the OMR Answer Sheet, except for the space allotted for the relevant entries, which may disclose your identity, you will render yourself liable to disqualification.
9. You have to return the test OMR Answer Sheet to the invigilators at the end of the examination compulsorily and must NOT carry it with you outside the Examination Hall.
10. You can take away question booklet and carbon copy of OMR Answer Sheet after the examination.
11. **Use only Blue/Black Ball point pen.**
12. **Use of any calculator, Electronic gadgets or log table etc., is prohibited.**
13. **There is no negative marks for incorrect answers.**
14. **In case of any discrepancy found in the Kannada translation of a question booklet the question in English version shall be taken as final.**

Total Number of Pages : 16

# MATHEMATICAL SCIENCE
## PAPER – III

**Note :** This paper contains **seventy-five (75)** objective type questions. **Each** question carries **two (2)** marks. **All** questions are **compulsory**.

**1.** Defining the value 0 at x = 0, which of the following functions is continuous on $[0, \pi]$ ?

(A) tanx (B) $\frac{1}{x}$

(C) $\sin\frac{1}{x}$ (D) $x\sin\frac{1}{x}$

**2.** Let $f : [a, b] \to \mathbb{R}$ take the value 1 at rational points and 0 at irrational points. Then

(A) f is Lebesgue integrable and $\int_a^b f = 0$

(B) f is Lebesgue integrable and $\int_a^b f = 1$

(C) f is Lebesgue integrable and $\int_a^b f = b - a$

(D) f is not Lebesgue integrable

**3.** The function $f(x) = x^2$ from $\mathbb{R}$ to $\mathbb{R}$ is

(A) Discontinuous on $\mathbb{R}$

(B) Discontinuous at some points of $\mathbb{R}$

(C) Continuous but not uniformly on $\mathbb{R}$

(D) Uniformly continuous on $\mathbb{R}$

**4.** Let $f : \mathbb{R} \setminus \{0\} \to \mathbb{R}$ be defined by $f(x) = x + \frac{1}{x}$. Pick out the true statement :

(A) f has a local maximum at 1 and the value is 2

(B) f has a local maximum at – 1 and the value is – 2

(C) f does not have a local maximum

(D) f does not have a local minimum

**5.** The sum of the infinite series

$$1+\frac{1}{3}-\frac{1}{2}+\frac{1}{5}+\frac{1}{7}-\frac{1}{4}+\frac{1}{9}+\frac{1}{11}-\frac{1}{6}+\ldots$$

is

(A) $\frac{3}{2}$ log2 (B) $\frac{2}{3}$ log2

(C) $\frac{4}{3}$ log2 (D) log2

**6.** Let $f_n = (0, 1) \to \mathbb{R}$ be defined by $f_n(x) = \frac{1}{1+nx}$. Then

(A) $f_n \to 0$ pointwise but not uniformly

(B) $f_n \to 0$ uniformly

(C) $\{f_n\}$ does not converge even pointwise

(D) $f_n \to 1$ pointwise

**7.** $\{a_n\}$ is a sequence of integers infinitely many of which are non-zero. In general, what can you say about the radius of convergence R of the power series $\sum a_n x^n$ ?

(A) $R \geq 1$

(B) $R = 1$

(C) $R \leq 1$

(D) $R < 1$

**8.** The improper integral $\int_0^1 \frac{\sec x}{x} dx$ is

(A) Convergent

(B) Divergent

(C) Oscillating

(D) Conditionally convergent

**9.** Let $A = \begin{bmatrix} 3 & -2 \\ 9 & -3 \end{bmatrix}$. Pick the correct statement.

(A) $A^2 - 9I = 0$

(B) $A^{-1} = \left(\frac{1}{9}\right) A$

(C) $A^2 + A + 9I = 0$

(D) $A^{-1} = \left(-\frac{1}{9}\right) A$

**10.** Let $A = \begin{bmatrix} 5 & 2 \\ 0 & k \end{bmatrix}$. If A satisfies the polynomial equation $x^2 - 7x + 10 = 0$, then k must be

(A) 0

(B) 1

(C) 2

(D) Any real number

**11.** Suppose A is a $n \times n$ real matrix and $k > 1$ is such that $A^k = 0$. Then which one of the following is correct ?

(A) $(I - A)^{-1}$ exists but $(I + A)^{-1}$ does not exist

(B) $(I + A)^{-1}$ exists but $(I - A)^{-1}$ does not exist

(C) $(I + A)^{-1}$ and $(I - A)^{-1}$ both do not exist

(D) $(I + A)^{-1}$ and $(I - A)^{-1}$ both exist

**12.** Let $P_2(\mathbb{R})$ be the vector space of all real polynomials of degree $\leq 2$ and let $S = \{x + 1, x^2 + x - 1, x^2 - x + 1\}$. If W denotes the linear span of S, then

(A) $W = P_2(\mathbb{R})$

(B) $W \neq P_2(\mathbb{R})$

(C) $W = \{0\}$

(D) Dimension of W is 1

**13.** If A is a $3 \times 3$ real matrix with det (A) = 6 and trace (A) = 0 such that $A + I$ is singular, then the possible eigenvalues of A are

(A) – 1, 2, 3

(B) – 1, 2, – 3

(C) 1, 2, – 3

(D) – 1, – 2, 3

**14.** Let $\Delta = \begin{vmatrix} 1 & t & t^2 \\ t & 1 & t \\ t^2 & t & 1 \end{vmatrix}$ and

$S = \{t \in \mathbb{C} : \Delta = 0\}$. Then which one of the following is correct ?

(A) S is a singleton

(B) S is a set with four elements

(C) S has infinitely many elements

(D) S is a set with two elements

**15.** Let $A = \begin{bmatrix} 4 & 1 & 2 & 2 \\ 0 & 3 & 2 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ and let N, R denote respectively the null space of A and the range space of A. Which one of the following is correct ?

(A) N = {0}

(B) $R = \mathbb{R}^3$

(C) R is a 3-dimensional subspace of $\mathbb{R}^4$

(D) N is a 2-dimensional subspace of $\mathbb{R}^4$

**16.** If $|z - 1| = 2$, the value of $z\bar{z} - z - \bar{z}$ is

(A) 3

(B) 5

(C) $|z|^2 - 2$

(D) $|z|^2 + 2$

**17.** Let S denote the set of all values of $i^i$ written in the form a + bi, where a and b are real. Then

(A) S = {1, i, – 1, – i}

(B) S = {0}

(C) $S = \left\{ e^{\pi/2}(1 + ki) : \text{k is any integer} \right\}$

(D) $S = \left\{ e^{-(\pi/2 + 2k\pi)} : \text{k is any integer} \right\}$

**18.** Suppose we write $i^{1/4}$ as $re^{i\theta}$ with $0 \le \theta \le \pi/2$. Then (r, $\theta$) must be

(A) $(1, \pi/8)$

(B) $(1, \pi/4)$

(C) $(1/2, \pi/8)$

(D) $(1/4, \pi/4)$

**19.** If we write cosz = u + iv, then u = 0 for

(A) Only $z = \pi/2$

(B) Only $z = \pi/2 + 2k\pi$ ; k any integer

(C) Infinitely many lines parallel to the y-axis : $x = \left(\frac{2k+1}{2}\right)\pi$; k any integer

(D) Infinitely many lines parallel to the x-axis : $y = \left(\frac{2k+1}{2}\right)\pi$; k any integer

**20.** Let $I = \int_0^{2\pi} e^{e^{i\theta}} d\theta$, the value of I is

(A) 1

(B) 0

(C) $2\pi i$

(D) $2\pi$

**21.** Let f be an analytic function on $|z| \leq R$.

Then $\int_{|z|=R} f(z)\,dz$

(A) is zero

(B) is a non zero complex number

(C) can be a non zero real number

(D) can be a non zero purely imaginary number

**22.** Let 'r' be real, $0 \leq r \leq 1$. The Laurent series given by $\sum_{n=-\infty}^{\infty} r^{|n|} e^{in\theta}$ is

(A) Convergent and converges to $\frac{1-r^2}{1-2r\cos\theta + r^2}$

(B) Convergent and converges to $\frac{1}{1-r e^{i\theta}}$

(C) Convergent and converges to $\frac{r}{1-r e^{i\theta}}$

(D) Convergent and converges to $\frac{1-r}{1+r}$

**23.** Let T be a Mobius transformation of $\mathbb{C}$. If T carries 0 to z in $\mathbb{C}$. Then

(A) T maps every circle in $\mathbb{C}$ to a circle in $\mathbb{C}$

(B) T maps every line in $\mathbb{C}$ to a line in $\mathbb{C}$

(C) T may carry some circle through 0 to a line through z

(D) T must carry every circle passing through 0 in $\mathbb{C}$ to a circle through z in $\mathbb{C}$

**24.** Which one of the following statements is false ?

(A) Any group of order 4 is cyclic

(B) Any group of order 121 is abelian

(C) Any group of order 163 is cyclic

(D) For any prime P, the multiplicative group of prime residue classes modulo P is cyclic

**25.** The number of conjugacy classes of the elements of the symmetric group $S_4$ on four letters is

(A) 4 (B) 24

(C) 5 (D) 12

**26.** Let $S_n$ be the symmetric (or permutation) group on n elements. Then which one of the following statements is true ?

(A) Every element of $S_n$ is a product of even permutations

(B) Every element of $S_n$ is a product of odd permutations

(C) Every even permutation has order 2

(D) Every odd permutation is a product of even permutations

**27.** In the ring of integers, the false statement in the following is

(A) Every ideal is principal

(B) Every non-zero prime ideal is maximal

(C) Every prime ideal is maximal

(D) Every maximal ideal is prime

**28.** Let $\mathbb{Z}[i]$ denote the ring of Gaussian integers and n be a positive integer. If $\mathbb{Z}[i] / n\mathbb{Z}[i]$ is an integral domain, then 'n' can be equal to

(A) 2 (B) 19

(C) 13 (D) 17

**29.** For a prime p, let F be a finite field with $p^n$ elements. Then the false statement in the following is

(A) $(F, +) \cong \mathbb{Z}/p\mathbb{Z} \oplus .... \oplus \mathbb{Z}/p\mathbb{Z}$ (n copies of $\mathbb{Z}/p\mathbb{Z}$)

(B) The multiplicative group of non zero elements of F is isomorphic to $\mathbb{Z}/(p^n - 1)\mathbb{Z}$

(C) $F \cong \mathbb{Z}/p^n\mathbb{Z}$

(D) The characteristic of F is p

**30.** Let $\mathbb{F}$ be a finite field. Then which one of the following statements is true ?

(A) $\mathbb{F}$ has to be isomorphic to $\mathbb{Z}/p\mathbb{Z}$ for some prime number

(B) $\mathbb{F}$ has to be isomorphic to $\mathbb{Z}/p^n\mathbb{Z}$ for some prime number and for some integer $n \geq 1$

(C) $\mathbb{F}$ is contained in a finite extension field of rational numbers

(D) $\mathbb{F}$ is not a subfield of field of complex numbers

**31.** Let K be an extension field over a field F. Then

(A) K is always a finite dimensional vector space over F

(B) K is infinite dimensional vector space over F if K is not algebraic extension of F

(C) K is an algebraic extension over F then K is a finite dimensional vector space over F

(D) K is an algebraic extension of F and F is a finite field then K is finite dimensional over F

**32.** Let f be a continuous function between the spaces mentioned in each of the following cases. Pick out the case in which f can be onto

(A) $f : [-1, 1] \to \mathbb{R}$

(B) $f : [-1, 1] \to Q \cap [-1, 1]$

(C) $f : \mathbb{R} \to [-1, 1]$

(D) $f : [-1, 1] \to Q$

**33.** Which one of the following sets is homeomorphic to the set $\{(x, y) : xy = 1\}$ in $\mathbb{R}^2$ ?

(A) $\{(x, y) : xy = 0\}$

(B) $\{(x, y) : x^2 - y^2 = 1\}$

(C) $\{(x, y) : x^2 + y^2 = 1\}$

(D) $\{(x, y) : x^2 + y^2 \le 1\}$

**34.** Which of the following subsets of $\mathbb{R}^2$ is compact ?

(A) $\{(x, y) : xy = 1\}$

(B) $\{(x, y) : x^2 + y^2 < 1\}$

(C) $\left\{(x, y) : \frac{x^2}{2} + \frac{y^2}{3} = 1\right\}$

(D) $\left\{(x, y) : \frac{x}{2} + \frac{y}{3} = 1\right\}$

**35.** The one point compactification of the complex plane $\mathbb{C}$ is

(A) $\mathbb{C}$ (B) $S^2$

(C) $\mathbb{R}^2$ (D) $\mathbb{R}^3$

**36.** Let $Y_1$ and $Y_2$ be two linearly independent solutions of

$\frac{d^2y}{dx^2} + (\sin x)\, y = 0$ in $0 \le x \le 1$. Let

$g(x) = W(Y_1, Y_2)(x)$ be the Wronskian of $Y_1$ and $Y_2$. Then

(A) $g'(x) > 0$ on $[0, 1]$

(B) $g'(x) < 0$ on $[0, 1]$

(C) $g'(x)$ vanishes at only one point of $[0, 1]$

(D) $g'(x)$ vanishes at all points of $[0, 1]$

**37.** The complete integral of the equation $2xz - px^2 - 2q\,xy + pq = 0$ is

(A) $z = ay + b\,(x^2 - a^2)$

(B) $z = ax + b\,(x^2 - a^2)$

(C) $z = bx - a\,(x^2 - a^2)$

(D) $z = by + a\,(x^2 - a^2)$

**38.** For the differential equation

$$t\,(t-2)^2\,\frac{d^2y}{dt^2}+t\,\frac{dy}{dt}+y=0\text{, the}$$

point t = 0 is

(A) an ordinary point

(B) an irregular point

(C) a regular singular point

(D) a branch point

**39.** Which of the following satisfies the heat equation (without source terms and diffusion constant one) in one space dimension ?

(A) $\sin(x^2/4t)$

(B) $\dfrac{e^{-x^2/4t}}{\sqrt{t}}$

(C) $x^2 - t$

(D) $e^t \sin x$

**40.** One root of the equation $e^x - 3x^2 = 0$ lies in the interval (3, 4). The least number of iterations required for the bisection method so that $|\text{error}| \le 10^{-3}$ is

(A) 8

(B) 10

(C) 6

(D) 4

**41.** The Runge-Kutta method of order four is used to solve the differential equation $\frac{dy}{dx}=f(x);\ y(0)=0$ with step size h. The solution at x = h is given by

(A) $y(h)=\frac{h}{6}\left[f(0)+4f\left(h/2\right)+f(h)\right]$

(B) $y(h)=\frac{h}{6}\left[f(0)+2f\left(h/2\right)+f(h)\right]$

(C) $y(h)=\frac{h}{6}\left[f(0)+f(h)\right]$

(D) $y(h)=\frac{h}{6}\left[2f(0)+f\left(h/2\right)+2f(h)\right]$

**42.** Consider the system of equations $x_1 - ax_2 = b_1;\ -ax_1 + x_2 = b_2$ where 'a' is a real constant. Jacobi's iteration scheme for solving these equations converges when

(A) $|a| \ge 1$ (B) $|a| > 1$

(C) $|a| < 1$ (D) $|a| \le 1$

**43.** The functional

$$I[y(x)] = \int_0^1 \left(xy + y^2 - 2y^2\frac{dy}{dx}\right)dx;$$

y(0) = 1, y(1) = 2 has

(A) no extremal

(B) one extremal

(C) infinite extremal

(D) exactly two extremals

**44.** Which one of the following functions is a solution of the Fredholm type integral equation $y(x) = x + \int_0^1 xt\ y(t)\,dt$

(A) $\frac{2}{3}x$ (B) $\frac{3}{2}x$

(C) $\frac{3}{4}x$ (D) $\frac{4}{3}x$

**45.** For the Hamiltonian $H = \frac{1}{2}\left(\frac{1}{q^2} + p^2q^4\right)$ the equation of motion for q can be written as $f(q, \dot{q}, \ddot{q}) = 0$ where $f(q, \dot{q}, \ddot{q})$ is

(A) $\ddot{q} + \frac{2\dot{q}^2}{q} - q$ (B) $\ddot{q} - \frac{2\dot{q}^2}{q} + q$

(C) $\ddot{q} - \frac{2\dot{q}^2}{q} - q$ (D) $q\,\ddot{q} - \dot{q}^2 - q^2$

**46.** In a frequency distribution, the first four central moments are; 0, 4, – 2 and 2.4. Then, what are the values of skewness $(\beta_1)$ and Kurtosis $(\beta_2)$ of the distribution

(A) $\beta_1 = 0.0625,\ \beta_2 = 0.15$

(B) $\beta_1 = 0.6250,\ \beta_2 = 0.15$

(C) $\beta_1 = 0.0625,\ \beta_2 = 0.20$

(D) $\beta_1 = 0.0690,\ \beta_2 = 0.151$

**47.** Let $\Omega$ = {a, b, c, d}. $\mathscr{F}$ is the minimal $\sigma$-field containing {{a, b}, {c}} . Which one of the following is not a member of $\mathscr{F}$ ?

(A) {a, b, c}

(B) {a, b, d, e}

(C) {a, b, c, d}

(D) {d, e}

**48.** In a club 40% are gents and 60% are ladies. 20% of gents are married and 80% of ladies are married. The probability that randomly selected person is married if that person is a lady.

(A) $1/7$

(B) $6/7$

(C) $3/4$

(D) $4/5$

**49.** Given that $P(B^C) = 0.4, P(A \cap B^c) = 0.3$ and $P(A^C \cap B) = 0.4$, then $P(A \cup B)$ is

(A) 0.7

(B) 0.9

(C) 0.8

(D) 0.2

**50.** Let X have the following probability distribution

$$F(x)=\begin{cases}0 & \text{if } x<0\\ \frac{1}{10} & \text{if } 0\le x<1\\ \frac{x^2}{20} & \text{if } 1\le x<4\\ 1 & \text{if } x\ge 4\end{cases}$$

(A) $\frac{109}{40}$

(B) $\frac{66}{30}$

(C) $\frac{63}{30}$

(D) $\frac{171}{60}$

**51.** If a r.v. X is uniformly distributed over (– 5, 5), then $P(|X|>2)$ is

(A) $\frac{4}{5}$ (B) $\frac{7}{8}$

(C) $\frac{3}{7}$ (D) $\frac{3}{5}$

**52.** Let X be uniform (– 1, 1) distribution, then the distribution of $Y = X^2$ over (0, 1) is

(A) $\frac{\sqrt{x}}{2}$ (B) $\frac{x^2}{2}$

(C) $x^2$ (D) $\sqrt{x}$

**53.** If X has geometric distribution with parameter p, then E(X) and Var (X) are

(A) $\left(\frac{1-p}{p}, \frac{(1-p)^2}{p}\right)$

(B) $\left(\frac{1-p}{p}, \frac{p}{1-p}\right)$

(C) $\left(\frac{1-p}{p}, \frac{1-p}{p^2}\right)$

(D) $\left(\frac{1-p}{p^2}, \frac{1-p}{p^2}\right)$

**54.** Let $Y = X_1 + X_2 + .... + X_{36}$ where $X_1, X_2, ... X_{36}$ are iid random variables with mean 2 and variance 1, then $P(Y < 83.76)$ is

(A) 0.95

(B) 0.05

(C) 0.025

(D) 0.975

**55.** Suppose a random variable X is such that $P(X = 1) = 1$ and $\phi(t)$ is its characteristic function then

(A) $EX = 1, V(X) = 0, \phi(t) = 1$ for all t

(B) $EX = 1, V(X) = 1, \phi(t) = 0$ for all t

(C) $EX = 1, V(X) = 0, \phi(t) = e^{it}$ for all t

(D) $EX = 0, V(X) = 1, \phi(t) = e^{it}$ for all t

**56.** If the mgf of a r.v. X is $\left(\frac{1}{3}+\frac{2}{3}e^t\right)^5$, then $P(X = 2 \text{ or } 3)$ is

(A) $\frac{40}{81}$

(B) $\frac{41}{81}$

(C) $\frac{39}{81}$

(D) $\frac{36}{81}$

**57.** The classification of the stochastic process in the case of daily rainfall measurement in an area is

(A) Discrete state continuous time process

(B) Discrete state discrete time process

(C) Continuous state continuous time process

(D) Continuous state discrete time process

**58.** Let $X_1, X_2, X_3$ be a sample of size 3 from a Bernoulli distribution. Which of the following is not a sufficient statistic ?

(A) $2(X_1 + X_2 + X_3)$

(B) $X_1 X_2 + X_3$

(C) $6(X_1, X_2 + X_3)$

(D) $\frac{1}{2}(X_1 + X_2, X_3)$

**59.** Let $X_1, X_2, ..., X_n$ be iid $B(1, \theta)$. Then, Bayes estimator of $\theta$ with respect to uniform prior distribution over (0, 1) is

(A) $\frac{\sum x_i}{n}$

(B) $\frac{\sum x_i + 1}{n}$

(C) $\frac{\sum x_i + 1}{n+2}$

(D) $\frac{\sum x_i}{n+2}$

**60.** Consider a r.v. X that binomial distribution with n = 3 and probability of success is p. Let f(x, p) denote the pmf of X and let $H_0 : p = \frac{1}{2}$ and $H_1 : p = \frac{3}{4}$. The following table gives the values of $f\left(x, \frac{1}{2}\right)$ and $f\left(x, \frac{3}{4}\right)$ :

| x | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $f\left(x, \frac{1}{2}\right)$ | $\frac{1}{8}$ | $\frac{3}{8}$ | $\frac{3}{8}$ | $\frac{1}{8}$ |
| $f\left(x, \frac{3}{4}\right)$ | $\frac{1}{64}$ | $\frac{9}{64}$ | $\frac{27}{64}$ | $\frac{27}{64}$ |

Based on one observation x, which one of the following critical regions leads to MP test of size $\alpha = \frac{1}{8}$ ?

(A) C. R. = $\{x : x = 0\}$

(B) C. R. = $\{x : x = 1\}$

(C) C. R. = $\{x : x > 3\}$

(D) C. R. = $\{x : x \geq 3\}$

**61.** Let $X_1, X_2, ..., X_n$ be a random sample from exponential population with pdf

$$f(x,\theta) = \begin{cases} e^{-(x-\theta)} & , x > \theta \\ 0 & , x < \theta \end{cases}$$

Then, the rejection region based on LRT procedure for testing $H_0 : \theta \le \theta_0$ vs $H_1 : \theta > \theta_0$ is : (here, C is a constant determined from size condition).

(A) $\left\{ \underline{X} : X_{(1)} \ge \theta_0 - \frac{\log C}{n} \right\}$

(B) $\left\{ \underline{X} : X_{(1)} \le \theta_0 + \frac{\log C}{n} \right\}$

(C) $\left\{ \underline{X} : e^{-n\left(X_{(1)}-\theta\right)} \ge C \right\}$

(D) $\left\{ \underline{X} : \left| e^{-n\left(X_{(1)}-\theta\right)} \right| > C \right\}$

**62.** For the simple linear regression model $y_i = \alpha + \beta x_i + \varepsilon_i$, i = 1 to n, the standard error of ordinary least squares estimator of $\beta$ with usual notation is

(A) $\sigma\sqrt{\frac{1}{n} + \frac{\bar{x}^2}{S_{xx}}}$ (B) $\frac{\sigma}{\sqrt{S_{xx}}}$

(C) $\sqrt{\frac{\text{M.S.E.}}{S_{xx}}}$ (D) $\sigma\sqrt{\frac{\bar{x}^2}{S_{xx}}}$

**63.** With reference to a Gauss-Markov model Y = $A\beta + \varepsilon$, which one of the following is not true ?

(A) The variance of the best estimate of estimable linear parametric function $\lambda'\beta$ is $\lambda'\mu\sigma^2$ where $A'A\mu = \lambda$.

(B) The linear parametric function $\lambda'\beta$ is estimable if $\lambda$ belongs to the estimation space of the linear model.

(C) The general mean $\mu$ is estimable.

(D) Using the normal equation $A'A\beta = A'Y$. $l'(A'A\beta)$ gives estimable parametric function and $l'(A'Y)$ given its best estimate.

**64.** Match List – I with List – II and select the correct answer using the codes below :

| List – I | List – II |
|---|---|
| a) Elimination of variation due to concomitant variables | 1) Incomplete block design |
| b) Handling large number of treatment combinations | 2) Analysis of covariance |
| c) Control of heterogeneity | 3) Local control |

(A) (a – 1), (b – 2), (c – 3)

(B) (a – 2), (b – 1), (c – 3)

(C) (a – 3), (b – 1), (c – 2)

(D) (a – 2), (b – 3), (c – 1)

**65.** The average variance of elementary treatment contrast in BIBD is

(A) $\frac{2r\sigma^2}{\lambda\nu}$ (B) $\frac{2k\sigma^2}{\nu r}$

(C) $\frac{2k\sigma^2}{\lambda\nu}$ (D) $\frac{2r}{bk}\sigma^2$

**66.** A 100 (1 – $\alpha$)% confidence limit on the mean response at the point $X = X_0$ for a simple linear regression model with n observation is

(A) $\hat{Y}_0 \pm t_{\alpha/2, n-2}\sqrt{\text{M.S.E.}\left(\frac{1}{n}+\frac{(X_0-\overline{X})^2}{S_{xx}}\right)}$

(B) $\hat{Y}_0 \pm t_{\alpha/2, n-2}\sqrt{\text{M.S.E.}\,\frac{(X_0-\overline{X})^2}{S_{xx}}}$

(C) $\hat{Y}_0 \pm t_{\alpha/2, n-2}\sqrt{\frac{\text{M.S.E.}}{S_{xx}}}$

(D) $\hat{Y}_0 \pm t_{\alpha/2, n-2}\sqrt{\text{M.S.E.}\left(\frac{1}{n}+\frac{\overline{X}^2}{S_{xx}}\right)}$

**67.** The dispersion matrix of residual vector $e = Y - \hat{Y}$ in a multiple linear regression $Y = X\beta + \varepsilon$, $\hat{Y} = HY$ is

(A) $H\sigma^2$ (B) $\sigma^2 I$

(C) $(I - H)\sigma^2$ (D) $(X'X)^{-1}\sigma^2$

**68.** If (X, Y) has bivariate normal density with $\mu_x = 10, \sigma_x^2 = 9, \mu_y = 12$, $\sigma_y^2 = 16$ and $\rho = 0.6$. Then the conditional density of Y given X = x is

(A) N (0.8 x + 4, 16 (1 – $0.6^2$))

(B) N (4 + x, 9 (1 – $0.6^2$))

(C) N (6 + 0.3 x, 4 (1 – $0.6^2$))

(D) N (12 + 0.8 x, 16 (1 – $0.4^2$))

**69.** Let $X \sim N_3(\mu, \Sigma)$ with $\mu = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$ and

$$\Sigma = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 2 \\ 1 & 2 & 2 \end{bmatrix}.$$

The distribution of $3X_1 - 2X_2 + X_3$ is

(A) Normal with mean 13 and variance 9

(B) Normal with mean 13 and variance 10

(C) Normal with mean 8 and variance 12

(D) Normal with mean 13 and variance 7

**70.** The following technique can be used to detect multivariate outliers

(A) Normal probability plot

(B) Box-plot

(C) Q-Q-plot

(D) Generalized square distance $(x_j - \overline{x})^1 S^{-1} (x_j - \overline{x})$

**71.** A sample of size n is drawn using SRSWOR (N, n) scheme from a dichotomous population. If the sample has proportion p of items of category I and proportion q of category II, then an unbiased estimate of variance of proportion p is

(A) $s_p^2 = \frac{npq}{(n-1)}$

(B) $s_p^2 = \frac{p^2 q}{(n-1)}$

(C) $s_p^2 = \frac{(N-n)}{N} p.\ q$

(D) $s_p^2 = \frac{(N-n)\, p.\ q}{N\,(n-1)}$

**72.** Under Lahiri sampling scheme, probability of selecting $i^{th}$ population unit is

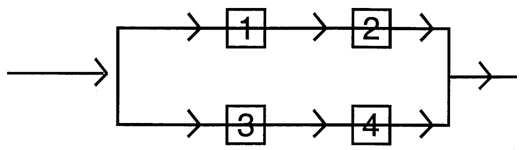
(A) $\frac{X_i}{X}$

(B) $\frac{n-1}{N-1} \cdot \frac{X_i}{X}$

(C) $\frac{n}{N}$

(D) $\frac{n}{N} \frac{X_i}{X}$

**73.** A system consist of four units connected as given below. The reliability of units 1, 2, 3 and 4 are respectively 0.6, 0.7, 0.8 and 0.6.




Then the reliability of the system is

(A) 0.90

(B) 0.67

(C) 0.50

(D) 0.70

**74.** In M/G/1 queuing system, G refers to

(A) Interarrival time is gamma distributed

(B) Service time is gamma distributed

(C) Interarrival time is general

(D) Service time is general

**75.** In standard (s, S) inventory system

(A) Demand is random

(B) Selling price is random

(C) Purchase price is random

(D) Set up cost is random

ಚಿತ್ತು ಬರಹಕ್ಕಾಗಿ ಸ್ಥಳ
Space for Rough Work

ಚಿತ್ತು ಬರಹಕ್ಕಾಗಿ ಸ್ಥಳ
Space for Rough Work