Date:

Registration number:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BENGALURU-27
B.Sc. MATHEMATICS - VI SEMESTER
SEMESTER EXAMINATION: APRIL 2022
(Examination conducted in July 2022)

**MT6218 – MATHEMATICS-VIII**

Time- 2 ½ hrs Max Marks-70

This question paper contains TWO printed pages and THREE parts

**I. Answer any FIVE of the following questions** **(5X2=10)**

1. Evaluate $\int_C \left[(3x+y)dx+(2y-x)dy\right]$ along the curve $y=3x+1$, from $(0,1)$ and $(3,10)$.
2. Evaluate $\int_0^1\int_0^1 x^2y^2\,dy\,dx$.
3. Evaluate $\int_0^1\int_1^3\int_1^2 dz\;dy\,dx$.
4. Write the statement of Stoke's theorem.
5. If $L[f(t)]=F(s)$, then prove that $L[e^{at}f(t)]=F(s-a)$.
6. Find the Laplace transform of $(1+t)^3$.
7. Find the Laplace transform of $[t\cosh(at)]$.
8. Show that $L\int_0^t\left[(t-u)u\,e^{-au}\,du\right]=\dfrac{1}{s^2(s+a)^2}$ .

**II. Answer any SEVEN of the following questions** **(7X6=42)**

9. Show that $\int_C\left[2xy\,dx+(x^2+2zy)\,dy+(y^2+1)\,dz\right]$ is path independent and hence evaluate, where $C$ be any path leading from the origin to the point $(1,1,1)$.
10. Evaluate $\iint_D xy(x+y)\,dy\,dx$ over the domain $D$ between $y=x$ and $y=x^2$.
11. Evaluate $\int_0^a\int_0^{2\sqrt{ax}} x^2\,dy\,dx$ by changing the order of integration.
12. Find the surface area of the cylinder $x^2+y^2=4$ cut by the cylinder $x^2+z^2=4$.

13. Evaluate $\int_{-a}^{a} \int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} \int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} dz\,dy\,dx$.

14. Find the volume bounded by the surface $z = a^2 - x^2$ and the planes $x = 0,\ y = 0,\ z = 0$ and $y = b$.

15. Verify Green's theorem in the plane for $\oint_C \left(xy + y^2\right)dx + x^2\,dy$ where $C$ is the closed curve bounded by $y = x$ and $y = x^2$.

16. State and Prove Gauss Divergence theorem.

17. Evaluate $\oint_C \sin z\,dx - \cos x\,dy + \sin y\,dz$ using Stoke's theorem where $C$ is the boundary of the rectangle $0 \le x \le \pi,\ 0 \le y \le 1,\ z = 3.$

**III.** **Answer any THREE of the following questions** **(3X6=18)**

18. Find the Laplace transform of $f(t) = \begin{cases} t & 0 < t < \pi \\ \pi - t & \pi < t < 2\pi \end{cases}$ with $f(t) = f(t + 2\pi)$.

19. If $L[f(t)] = F(s)$ then prove that $L\left[\frac{f(t)}{t}\right] = \int_s^{\infty} F(s)\,ds$.

20. Find the inverse Laplace transform of $\frac{1}{s^2\left(s^2 + 1\right)}$.

21. Verify the convolution theorem for $f(t) = e^t$ and $g(t) = \cos t$.

22. Solve $9y'' - 6y' + y = 0$ using Laplace transform, where $y(0) = 3$ and $y'(0) = 1$.

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